Additional data file 1

A brief overview of slender body theory used for the calculation of flow around a pair of slender bodies.

A brief summary of slender body theory (based on chapter 7 of [25]) and its application to the two-body problem is presented here. We start by defining a slender body as having an elongate shape, such that $d/l << 1$, where d is the maximum diameter and l the animal length. As a result, flow characteristics do not change rapidly in the longitudinal (x) direction. The flow field around it can thus be assumed to be a series of two-dimensional flow fields around each cross-section [25] only slightly differing from each other. This model has been shown to be sufficiently accurate for ratios of up to 0.4 (see page 187 of [31]). In our case, the slenderness ratio is 1/6, well within the range of the model's validity.

To simplify matters we only look at bodies of revolution, such that the cross-sectional area is $S = \pi r^2$, where r is the local cross-section radius. For motion in the direction of the longitudinal axis, the only change in the flow field is due to changes in the local radius, and the flow around the whole body can be described by the flow around a row of doublet-type singularities at the axis (see page 213 of [10]). As a result, a single symmetric closed body such as our spheroid will experience zero net force (the D'Alembert paradox) in inviscid flow. The drag force in this case is purely of viscous origin and is not included in this type of model. Therefore, the ellipsoid shown in Figure 3 will experience zero net force, and all we can gain from that calculation is an understanding of the flow field. However, when two bodies are in proximity, the flow asymmetries caused result in net forces on each of them.

The analysis of two adjacent slender bodies closely follows [18,19], as mentioned in the text. Cartesian coordinate systems attached to the centroids of both bodies, where index 1 will indicate the mother and 2 the calf. These coordinate systems are moving at speeds U_1 and U_2, respectively, in the x direction. It is useful to relate both to a fixed coordinate system so that

$$x_0 = x_1 + U_1 t = x_2 + U_2 t - \xi(0) \qquad \text{(A-1)}$$

$$y_0 = y_1 = y_2 - \eta$$

$$z_0 = z_1 = z_2$$

Note that $\xi(t) = \xi(0) + (U_1 - U_2)t$ is the stagger between the bodies.

Potential incompressible flow is described by the linear Laplace equation $\nabla^2\varphi = 0$, where φ is the flow potential, such that its derivative in any direction is the velocity in that direction $\frac{\partial \varphi}{\partial n} = u_n$.

The Laplace equation is linear, so that solutions can be superposed. As mentioned above, the flow field around circular cross-sections is defined by a doublet so that each of the bodies when immersed in a uniform stream of speed U_i is described by a row of doublets of strength d_i, where S_i is the cross-sectional area

$$d_i(x_i) = -\frac{1}{2\pi} S_i(x_i) U_i \qquad i = 1,2 \qquad \text{(A-2)}$$

The potential of the flow can now be written as a sum of the potentials of flow around each body

$$\varphi(x_1,y_1) = -U_1 x_1 + \varphi_1(x_1,y_1) + \varphi_2(x_2,y_2) + \text{smaller secondary interaction terms} \qquad \text{(A-3)}$$

Substituting equation (A-2) into (A-3) and applying the coordinate system of Figure 4, we can write the potential for the mother as

$$\varphi_2(x_2, y_2, z_2, \xi) = -\frac{U_2}{2\pi} \int_{L_{2i}} \frac{S_2(x_2)(x - x_2 + \xi)dx}{((x - x_2 + \xi)^2 + (y - \eta)^2 + z^2)^{\frac{3}{2}}} \qquad \text{(A-4)}$$

with a similar expression for the calf, where both ξ and η are zero and indices are 1.

We obtain the velocity at each point due to the mother by taking the suitable derivative of the potential. Specifically, on the calf axis the velocity components are

$$U(x_1, \xi) = \frac{\partial \varphi_2}{\partial x}(x_1, 0, 0, \xi) = \frac{U_2}{2\pi} \int_{L_{2i}} \frac{S_2^{'}(x_2)(x - x_2 - \xi)dx_2}{((x - x_2 - \xi)^2 + \eta^2)^{\frac{3}{2}}} \qquad \text{(A-5)}$$

and

$$V(x_1, \xi) = \frac{\partial \varphi_2}{\partial y}(x_1, 0, 0, \xi) = \frac{U_2 \eta}{2\pi} \int_{L_{2i}} \frac{S_2^{'}(x_2)dx_2}{((x - x_2 - \xi)^2 + \eta^2)^{\frac{3}{2}}} \qquad \text{(A-6)}$$

These velocities induce additional singularities on the axis of the calf, which can be written, again assuming slender bodies, as

$\mu_{1x}(x_1,\xi) = -\frac{1}{2\pi} S_1(x_1)U(x_1,\xi)$ and $\mu_{1y}(x_1,\xi) = -\frac{1}{2\pi} S_1(x_1)V(x_1,\xi)$, respectively.

These singularities enable calculation of the forces on the calf resulting from the interaction, as mentioned previously. These are, to leading order,

$$F_x(\xi) = -\rho U_2 \int S_1(x_1)\frac{\partial}{\partial \xi} U dx_1 \quad \text{and} \quad F_y(\xi) = -2\rho U_2 \int S_1(x_1)\frac{\partial}{\partial \xi} V dx_1 \tag{A-7}$$

Next, we integrate by parts and apply the fact that the body tips are mathematically smooth, to obtain

$$X = F_x = \frac{\rho U_2^2}{2\pi} \int_{L_1} S_1'(x_1) \int_{L_2} \frac{S_2'(x_2)(x_2 - x_1 - \xi)dx_2}{[(x_2 - x_1 - \xi)^2 + \eta^2]^{\frac{3}{2}}} dx_1 \tag{A-8}$$

and

$$Y = F_y = \frac{\rho U_2^2 \eta}{\pi} \int_{L_1} S_1'(x_1) \int_{L_2} \frac{S_2'(x_2)dx_2}{[(x_2 - x_1 - \xi)^2 + \eta^2]^{\frac{3}{2}}} dx_1 \tag{A-9}$$

for the longitudinal, and lateral forces on the calf due to the existence of the mother.

To apply the general equations (A-8) and (A-9) one now needs a specific body shape.